Chapter
Work Energy and Power

Topic-1: Work Done by Constant & Variable Force

1 MCQs with One Correct Answer

1. The work done on a particle of mass m by a force,

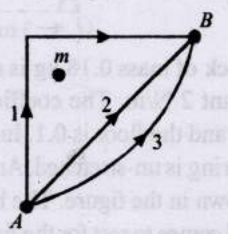
$$K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i}+\frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$$

(K being a constant of appropriate dimensions), when the particle is taken from the point $(a, 0)$ to the point $(0, a)$ along a circular path of radius a about the origin in the $x-y$ plane is **[Adv. 2013]**

(a) $\frac{2K\pi}{a}$ (b) $\frac{K\pi}{a}$

(c) $\frac{K\pi}{2a}$ (d) 0

2. If W_1, W_2 and W_3 represent the work done in moving a particle from A to B along three different paths 1,2 and 3 respectively (as shown) in the gravitational field of a point mass m, find the correct relation between W_1, W_2 and W_3 **[2003S]**



(a) $W_1 > W_2 > W_3$

(b) $W_1 = W_2 = W_3$

(c) $W_1 < W_2 < W_3$

(d) $W_2 > W_1 > W_3$

3. A force $F = -K(y\hat{i} + x\hat{j})$ (where K is a positive constant) acts on a particle moving in the xy plane. Starting from the origin, the particle is taken along the positive x axis to the point $(a, 0)$, and then parallel to the y axis to the point (a, a). The total work done by the force F on the particle is **[1998S - 2 Marks]**

(a) $-2Ka^2$ (b) $2Ka^2$ (c) $-Ka^2$ (d) Ka^2

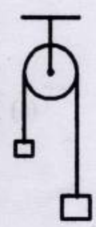
4. A uniform chain of length L and mass M is lying on a smooth table and one third of its length is hanging vertically down over the edge of the table. If g is acceleration due to gravity, the work required to pull the hanging part on to the table is **[1985 - 2 Marks]**

(a) MgL (b) $MgL/3$

(c) $MgL/9$ (d) $MgL/18$

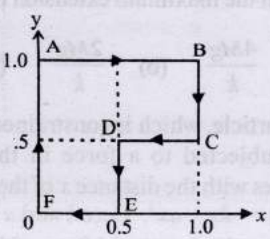
2 Integer Value Answer

5. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10$ m/s^2, find the work done **(in joules)** by the string on the block of mass 0.36 kg during the first second after the system is released from rest. **[2009]**



3 Numeric Answer/ New Stem Based Questions

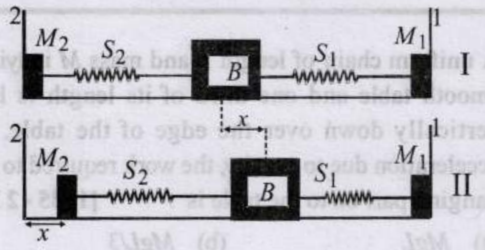
6. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a forced $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})N$, where x and y are in meter and $\alpha = -1Nm^{-1}$. The work done on the particle by this force $\vec{F}$ will be ____ Joule. **[Adv. 2019]**



Topic-2: Energy

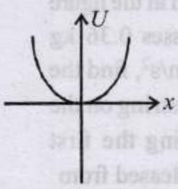
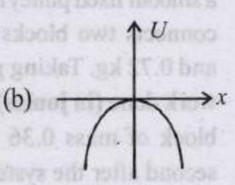
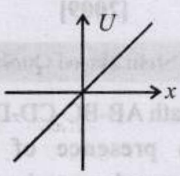
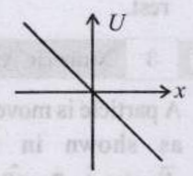
1 MCQs with One Correct Answer

1. A block (B) is attached to two unstretched springs S_1 and S_2 with spring constants k and $4k$, respectively (see fig. I). The other ends are attached to identical supports M_1 and M_2 not attached to the walls. The springs and supports have negligible mass. There is no friction anywhere. The block B is displaced towards wall 1 by a small distance x (figure II) and released. The block returns and moves a maximum distance y towards wall 2. Displacements x and y are measured with respect to the equilibrium position of the block B. The ratio y/x is – **[2008]**



(a) 4 (b) 2 (c) 1/2 (d) 1/4

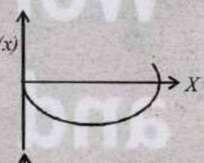
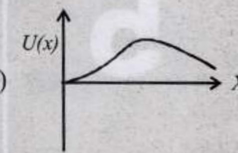
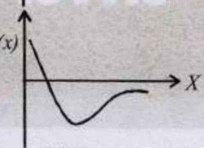
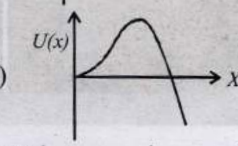
2. A particle is acted by a force $F = kx$, where k is a +ve constant. Its potential energy at $x = 0$ is zero. Which curve correctly represents the variation of potential energy of the block with respect to x **[2004S]**

(a) 
(b) 
(c) 
(d) 

3. An ideal spring with spring-constant k is hung from the ceiling and a block of mass M is attached to its lower end. The mass is released with the spring initially unstretched. Then the maximum extension in the spring is **[2002S]**

(a) $\frac{4Mg}{k}$ (b) $\frac{2Mg}{k}$ (c) $\frac{Mg}{k}$ (d) $\frac{Mg}{2k}$

4. A particle, which is constrained to move along the x-axis, is subjected to a force in the same direction which varies with the distance x of the particle from the origin as $F(x) = -kx + ax^3$. Here k and a are positive constants. For $x \geq 0$, the functional form of the potential energy $U(x)$ of the particle is **[2002S]**

(a) 
(b) 
(c) 
(d) 

5. A spring of force-constant k is cut into two pieces such that one piece is double the length of the other. Then the long piece will have a force-constant of **[1999S - 2 Marks]**

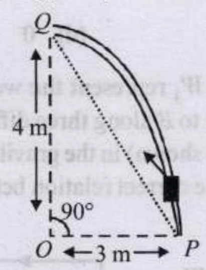
(a) $(2/3)k$ (b) $(3/2)k$ (c) $3k$ (d) $6k$

6. Two masses of 1 gm and 4 gm are moving with equal kinetic energies. The ratio of the magnitudes of their linear momenta is **[1980]**

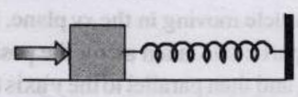
(a) $4:1$ (b) $\sqrt{2}:1$ (c) $1:2$ (d) $1:16$

2 Integer Value Answer

7. Consider an elliptical shaped rail PQ in the vertical plane with $OP = 3$ m and $OQ = 4$ m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictionless losses, the kinetic energy of the block when it reaches Q is $(n \times 10)$ joules. The value of n is (take acceleration due to gravity = 10 ms^{-2}) **[Adv. 2014]**



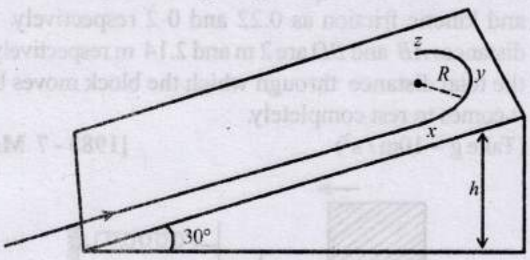
8. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V = N/10. Then N is **[2011]**



6 MCQs with One or More than One Correct Answer

9. A student skates up a ramp that makes an angle 30° with the horizontal. He/she starts (as shown in the figure) at the bottom of the ramp with speed v_0 and wants to turn around over a semicircular path xyz of radius R during which he/

she reaches a maximum height h (at point y) from the ground as shown in the figure. Assume that the energy loss is negligible and the force required for this turn at the highest point is provided by his/her weight only. Then (g is the acceleration due to gravity) **[Adv. 2020]**



(a) $v_0^2 - 2gh = \frac{1}{2}gR$

(b) $v_0^2 - 2gh = \frac{\sqrt{3}}{2}gR$

(c) the centripetal force required at points x and z is zero

(d) the centripetal force required is maximum at points x and z

10. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true? **[Adv. 2018]**

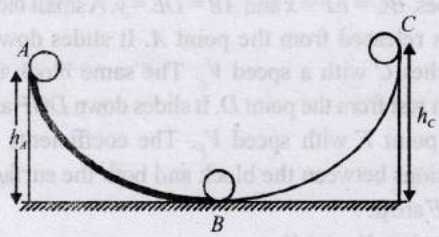
(a) The force applied on the particle is constant

(b) The speed of the particle is proportional to time

(c) The distance of the particle from the origin increases linearly with time

(d) The force is conservative

11. A small ball starts moving from A over a fixed track as shown in the figure. Surface AB has friction. From A to B the ball rolls without slipping. Surface BC is frictionless. K_A, K_B and K_C are kinetic energies of the ball at A, B and C, respectively. Then **[2006 - 5M, –1]**



(a) $h_A > h_C$; $K_B > K_C$ (b) $h_A > h_C$; $K_C > K_A$

(c) $h_A = h_C$; $K_B = K_C$ (d) $h_A < h_C$; $K_B > K_C$

12. A stone tied to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position, and has a speed u. The magnitude of the change in its velocity as it reaches a position where the string is horizontal is **[1998S - 2 Marks]**

(a) $\sqrt{u^2 - 2gL}$ (b) $\sqrt{2gL}$

(c) $\sqrt{u^2 - gL}$ (d) $\sqrt{2(u^2 - gL)}$

13. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane. It follows that : **[1987 - 2 Marks]**

(a) its velocity is constant

(b) its acceleration is constant

(c) its kinetic energy is constant.

(d) it moves in a circular path.

7 Match the Following

14. A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 constants). Match the potential energies in column I to the corresponding statement(s) in column II.

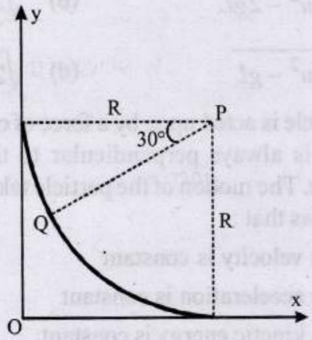
Column I	Column II
(A) $U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(p) The force acting on the particle is zero at $x = a$
(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(q) The force acting on the particle is zero at $x = 0$
(C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(r) The force acting on the particle is zero at $x = -a$
(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(s) The particle experiences an attractive force towards $x = 0$ in the region $\lvert x \rvert < a$
	(t) The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x = -a$

8 Comprehension Passage Based Questions

PASSAGE

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure below, is 150 J.

(Take the acceleration due to gravity, $g = 10\ ms^{-2}$) **[Adv. 2013]**



15. The magnitude of the normal reaction that acts on the block at the point Q is

(a) 7.5 N (b) 8.6 N

(c) 11.5 N (d) 22.5 N

16. The speed of the block when it reaches the point Q is

(a) $5\ ms^{-1}$ (b) $10\ ms^{-1}$

(c) $10\sqrt{3}\ ms^{-1}$ (d) $20\ ms^{-1}$

9 Assertion and Reason Type Questions

17. **STATEMENT–1 :** A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation.

STATEMENT–2 : The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination. **[2007]**

(a) Statement–1 is True, Statement–2 is True; Statement–2 is a correct explanation for Statement–1

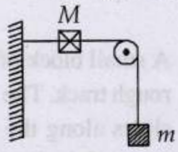
(b) Statement–1 is True, Statement–2 is True; Statement–2 is NOT a correct explanation for Statement–1

(c) Statement–1 is True, Statement–2 is False

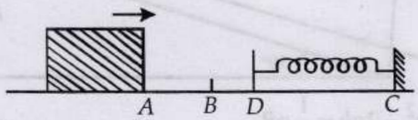
(d) Statement–1 is False, Statement–2 is True

10 Subjective Problems

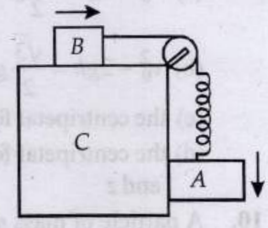
18. A string, with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2m from the wall, has a point mass $M = 2$kg attached to it at a distance of 1m from the wall. A mass $m = 0.5$ kg attached at the free end is held at rest so that the string is horizontal between the wall and the pulley and vertical beyond the pulley. What will be the speed with which the mass M will hit the wall when the mass m is released? **[1985 - 6 Marks]**



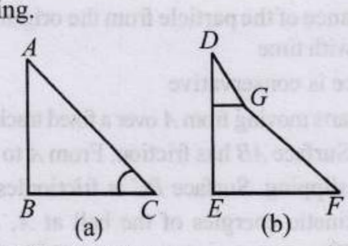
19. A 0.5 kg block slides from the point A (see Fig) on a horizontal track with an initial speed of 3 m/s towards a weightless horizontal spring of length 1 m and force constant 2 Newton/m. The part AB of the track is frictionless and the part BC has the coefficients of static and kinetic friction as 0.22 and 0 2 respectively. If the distances AB and BD are 2 m and 2.14 m respectively, find the total distance through which the block moves before it comes to rest completely.
(Take $g = 10 m/s^2$) **[1983 - 7 Marks]**



20. Two blocks A and B are connected to each other by a string and a spring; the string passes over a frictionless pulley as shown in the figure. Block B slides over the horizontal top surface of a stationary block C and the block A slides along the vertical side of C, both with the same uniform speed. The coefficient of friction between the surfaces of blocks is 0.2. Force constant of the spring is 1960 newtons/m. If mass of block A is 2 Kg., calculate the mass of block B and the energy stored in the spring. **[1982 - 5 Marks]**



21.



In the figures (a) and (b) AC, DG and GF are fixed inclined planes, $BC = EF = x$ and $AB = DE = y$. A small block of mass M is released from the point A. It slides down AC and reaches C with a speed V_c. The same block is released from rest from the point D. It slides down DGF and reaches the point F with speed V_F. The coefficients of kinetic frictions between the block and both the surface AC and DGF are μ. **[1980]**

Calculate V_C and V_F.

22. When a ball is thrown up, the magnitude of its momentum decreases and then increases. Does this violate the conservation of momentum principle? **[1979]**

23. A spring of force constant k is cut into three equal parts. What is force constant of each part? **[1978]**

24. A bullet is fired from a rifle. If the rifle recoils freely, determine whether the kinetic energy of the rifle is greater than, equal or less than that of the bullet. **[1978]**

Topic-3: Power

1 MCQs with One Correct Answer

1. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as $a_c = k^2rt^2$ where k is a constant. The power delivered to the particles by the force acting on it is: **[1994 - 1 Mark]**

(a) $2\pi\, mk^2r^2\, t$ (b) mk^2r^2t

(c) $\frac{(mk^4r^2t^5)}{3}$ (d) zero

2. If a machine is lubricated with oil **[1980]**

(a) the mechanical advantage of the machine increases.
(b) the mechanical efficiency of the machine increases.
(c) both its mechanical advantage and efficiency increase.
(d) its efficiency increases, but its mechanical advantage decreases.

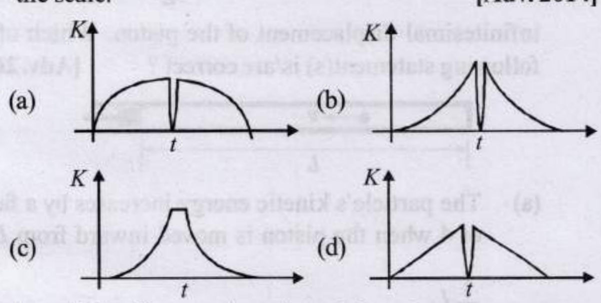
2 Integer Value Answer

3. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5 W to the particle. If the initial speed (in ms^{-1}) of the particle is zero, the speed (in ms^{-1}) after 5 s is **[Adv. 2013]**

Topic-4: Collisions

1 MCQs with One Correct Answer

1. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately? The figure are only illustrative and not to the scale. **[Adv. 2014]**



2. A particle of mass m is projected from the ground with an initial speed u_0 at an angle α with the horizontal. At the highest point of its trajectory, it makes a completely inelastic collision with another identical particle, which was thrown vertically upward from the ground with the same initial speed u_0. The angle that the composite system makes with the horizontal immediately after the collision is **[Adv. 2013]**

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{4}+\alpha$ (c) $\frac{\pi}{2}-\alpha$ (d) $\frac{\pi}{2}$

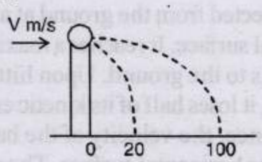
3. This question has statement I and statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **[2012]**

Statement - I: A point particle of mass m moving with speed υ collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f=\left(\frac{m}{M+m}\right)$.

Statement - II: Maximum energy loss occurs when the particles get stuck together as a result of the collision.

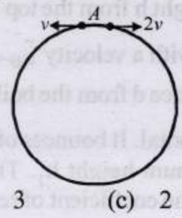
(a) Statement - I is true, Statment - II is true, Statement - II is the correct explanation of Statement - I.
(b) Statement-I is true, Statment - II is true, Statement - II is **not** the correct explanation of Statement - II.
(c) Statement - I is true, Statment - II is false.
(d) Statement - I is false, Statment - II is true.

4. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The velocity V of the bullet is **[2011]**



(a) 250 m/s (b) $250\sqrt{2}$ m/s
(c) 400 m/s (d) 500 m/s

5. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and $2v$, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A? **[2009]**



(a) 4 (b) 3 (c) 2 (d) 1

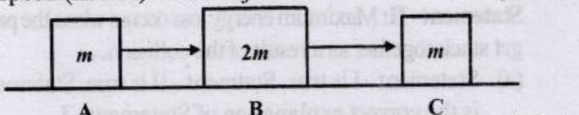
6. Two particles of masses m_1 and m_2 in projectile motion have velocities $\vec{v}_1$ and $\vec{v}_2$ respectively at time $t = 0$. They collide at time t_o. Their velocities become $\vec{v}_1\,'$ and $\vec{v}_2\,'$ at time $2t_o$ while still moving in air. The value of $\left|(m_1\vec{v}_1\,' + m_2\vec{v}_2\,') - (m_1\vec{v}_1 + m_2\vec{v}_2)\right|$ is **[2001S]**

(a) zero (b) $(m_1 + m_2)gt_o$

(c) $\frac{1}{2}(m_1 + m_2)gt_o$ (d) $2(m_1 + m_2)gt_o$

2 Integer Value Answer

7. Three objects *A*, *B* and *C* are kept in a straight line on a frictionless horizontal surface. These have masses *m*, *2m* and *m*, respectively. The object *A* moves towards *B* with a speed 9 m/s and makes an elastic collision with it. There after, *B* makes completely inelastic collision with *C*. All motions occur on the same straight line. Find the final speed (**in m/s**) of the object C. **[2009]**



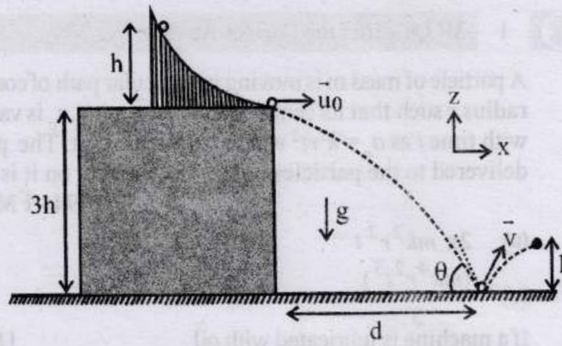
8. A bob of mass *m*, suspended by a string of length l_1, is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass *m* suspended by a string of length l_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum speed required to complete a full circle in the vertical plane, the ratio $\frac{l_1}{l_2}$ is **[Adv. 2013]**

3 Numeric Answer

9. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 *m* and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in *metres*, is ________. **[Adv. 2018]**

6 MCQs with One or More than One Correct Answer

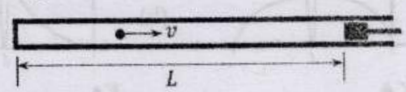
10. A slide with a frictionless curved surface, which becomes horizontal at its lower end, is fixed on the terrace of a building of height 3h from the ground, as shown in the figure. A spherical ball of mass m is released on the slide from rest at a height h from the top of the terrace. The ball leaves the slide with a velocity $\vec{u}_0 = u_0\hat{x}$ and falls on the ground at a distance d from the building making an angle θ with the horizontal. It bounces off with a velocity $\vec{v}$ and reaches a maximum height h_1. The acceleration due to gravity is g and the coefficient of restitution of the ground is $1/\sqrt{3}$. Which of the following statement(s) is(are) correct? **[Adv. 2023]**



(a) $\vec{u}_0 = \sqrt{2gh}\,\hat{x}$ (b) $\vec{v} = \sqrt{2gh}\left(\hat{x} - \hat{z}\right)$

(c) $\theta = 60°$ (d) $d/h_1 = 2\sqrt{3}$

11. A small particle of mass *m* moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed *V* such that $V \ll \frac{dL}{L}v_0$, where *dL* is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct ? **[Adv. 2019]**



(a) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

(b) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

(c) The rate at which the particle strikes the piston is *v*/L

(d) After each collision with the piston, the particle speed increases by 2 V.

12. A flat plate is moving normal to its plane through a gas under the action of a constant force F. The gas is kept at a very low pressure. The speed of the plate v is much less than the average speed u of the gas molecules. Which of the following options is/are true? **[Adv. 2017]**

(a) The pressure difference between the leading and trailing faces of the plate is proportional to uv

(b) The resistive force experienced by the plate is proportional to v

(c) The plate will continue to move with constant non-zero acceleration, at all times

(d) At a later time the external force F balances the resistive force

13. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. After their collision, the 1 kg mass reverses its direction and moves with a speed of 2 ms^{-1}. Which of the following statement(s) is (are) correct for the system of these two masses? **[2010]**

(a) Total momentum of the system is 3 kg ms^{-1}

(b) Momentum of 5 kg mass after collision is 4 kg ms^{-1}

(c) Kinetic energy of the centre of mass is 0.75 J

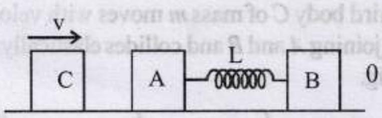
(d) Total kinetic energy of the system is 4J

14. The balls, having linear momenta $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p}_1'$ and $\vec{p}_2'$ be their final momenta. The following option (s) is (are) NOT ALLOWED for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2. **[2008]**

(a) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i} + b_2\hat{j}$ (b) $\vec{p}_1' = c_1\hat{k}$, $\vec{p}_2' = c_2\hat{k}$

(c) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}_2' = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$ (d) $\vec{p}_1' = a_1\hat{i} + b_1\hat{j}$, $\vec{p}_2' = a_2\hat{i} + b_1\hat{j}$

15. Two blocks *A* and *B*, each of mass *m*, are connected by a massless spring of natural length *L* and spring constant *K*. The blocks are initially resting on a smooth horizontal floor with the spring at its natural length, as shown in fig.. A third identical block *C*, also of mass *m*, moves on the floor with a speed *v* along the line joining *A* and *B*, and collides elastically with *A*. Then **[1993-2 Marks]**



(a) the kinetic energy of the *A-B* system, at maximum compression of the spring, is zero.

(b) the kinetic energy of the *A-B* system, at maximum compression of the spring, is $mv^2/4$.

(c) the maximum compression of the spring is $v\sqrt{(m/K)}$

(d) the maximum compression of the spring is $v\sqrt{(m/2K)}$

16. A shell is fired from a cannon with a velocity *v* (m/sec.) at an angle θ with the horizontal direction. At the highest point in its path it explodes into two pieces of equal mass. One of the pieces retraces its path to the cannon and the speed (in m/sec.) of the other piece immediately after the explosion is **[1986 - 2 Marks]**

(a) $3v\cos\theta$ (b) $2v\cos\theta$

(c) $\frac{3}{2}v\cos\theta$ (d) $\sqrt{\frac{3}{2}}v\cos\theta$

17. A ball hits the floor and rebounds after an inelastic collision. In this case **[1986 - 2 Marks]**

(a) the momentum of the ball just after the collision is the same as that just before the collision.

(b) the mechanical energy of the ball remains the same in the collision

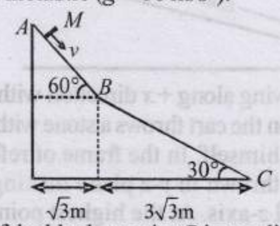
(c) the total momentum of the ball and the earth is conserved

(d) the total energy of the ball and the earth is conserved

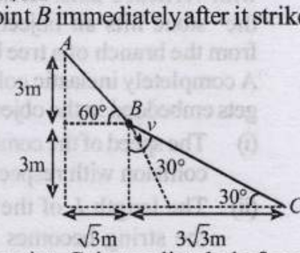
8 Comprehension/Passage Based Questions

PASSAGE

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point *B*. The block is initially at rest at *A*. Assume that collisions between the block and the incline are totally inelastic ($g = 10$ m/s^2). **[2008]**



18. The speed of the block at point *B* immediately after it strikes the second incline is –



(a) $\sqrt{60}$ m/s

(b) $\sqrt{45}$ m/s

(c) $\sqrt{30}$ m/s

(d) $\sqrt{15}$ m/s

19. The speed of the block at point *C*, immediately before it leaves the second incline is

(a) $\sqrt{120}$ m/s (b) $\sqrt{105}$ m/s (c) $\sqrt{90}$ m/s (d) $\sqrt{75}$ m/s

20. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point *B*, immediately after it strikes the second incline is –

(a) $\sqrt{30}$ m/s (b) $\sqrt{15}$ m/s (c) 0 (d) $-\sqrt{15}$ m/s

9 Assertion and Reason Type Questions

21. **STATEMENT–1 :** In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision. **[2007]**

STATEMENT–2 : In an elastic collision, the linear momentum of the system is conserved.

(a) Statement–1 is True, Statement–2 is True; Statement–2 is a correct explanation for Statement–1.

(b) Statement–1 is True, Statement–2 is True; Statement–2 is NOT a correct explanation for Statement–1

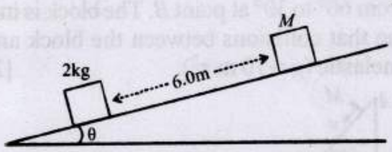
(c) Statement–1 is True, Statement–2 is False.

(d) Statement–1 is False, Statement–2 is True.

10 Subjective Problems

22. Two blocks of mass 2 kg and M are at rest on an inclined plane and are separated by a distance of 6.0 m as shown in Figure. The coefficient of friction between each of the blocks and the inclined plane is 0.25. The 2 kg block is given a velocity of 10.0 m/s up the inclined plane. It collides with M, comes back and has a velocity of 1.0 m/s when it reaches its initial position. The other block M after the collision moves 0.5 m up and comes to rest. Calculate the coefficient of restitution between the blocks and the mass of the block M.

[Take $\sin\theta \approx \tan\theta = 0.05$ and $g = 10\,m/s^2$.] **[1999 - 10 Marks]**

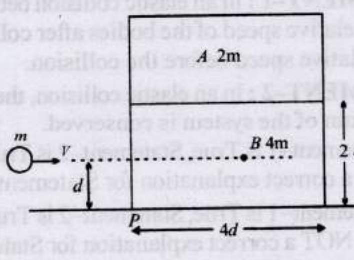


23. A cart is moving along $+x$ direction with a velocity of 4 m/s. A person on the cart throws a stone with a velocity of 6 m/s relative to himself. In the frame of reference of the cart the stone is thrown in y-z plane making an angle of 30° with vertical z-axis. At the highest point of its trajectory, the stone hits an object of equal mass hung vertically from the branch of a tree by means of a string of length L. A completely inelastic collision occurs, in which the stone gets embedded in the object. Determine : **[1997 - 5 Marks]**

(i) The speed of the combined mass immediately after the collision with respect to an observer on the ground,

(ii) The length L of the string such that the tension in the string becomes zero when the string becomes horizontal during the subsequent motion of the combined mass.

24. A block 'A' of mass 2m is placed on another block 'B' of mass 4m which in turn is placed on a fixed table. The two blocks have a same length $4d$ and they are placed as shown in fig. The coefficient of friction (both static and kinetic) between the block 'B' and table is μ. There is no friction between the two blocks. A small object of mass m moving horizontally along a line passing through the centre of mass (cm.) of the block B and perpendicular to its face with a speed v collides elastically with the block B at a height d above the table. **[1991 - 4 + 4 Marks]**

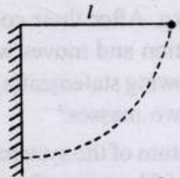


(a) What is the minimum value of v (call it v_0) required to make the block A topple ?

(b) If $v = 2v_0$, find the distance (from the point P in the figure) at which the mass m falls on the table after collision. (Ignore the role of friction during the collision).

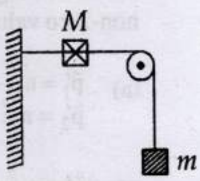
25. A simple pendulum is suspended from a peg on a vertical wall. The pendulum is pulled away from the wall to a horizontal position (see fig.) and released. The ball hits the wall, the coefficient of restitution being $\frac{2}{\sqrt{5}}$.

[1987 - 7 Marks]



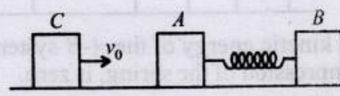
What is the minimum number of collisions after which the amplitude of oscillations becomes less than 60 degrees ?

26. A string, with one end fixed on a rigid wall, passing over a fixed frictionless pulley at a distance of 2m from the wall, has a point mass $M = 2$kg attached to it at a distance of 1m from the wall. A mass $m = 0.5$ kg attached at the free end is held at rest so that the string is horizontal between the wall and the pulley and vertical beyond the pulley. What will be the speed with which the mass M will hit the wall when the mass m is released? **[1985 - 6 Marks]**



27. A ball of mass 100 gm is projected vertically upwards from the ground with a velocity of 49 m/sec. At the same time another identical ball is dropped from a height of 98 m to fall freely along the same path as that followed by the first ball. After some time the two balls collide and stick together and finally fall to the ground. Find the time of flight of the masses. **[1985 - 8 Marks]**

28. Two bodies A and B of masses m and 2 m respectively are placed on a smooth floor. They are connected by a spring. A third body C of mass m moves with velocity v_0 along the line joining A and B and collides elastically with A as shown in Fig.



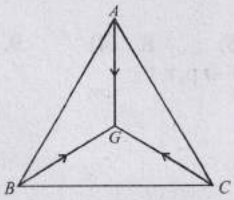
At a certain instant of time t_0 after collision, it is found that the instantaneous velocities of A and B are the same. Further at this instant the compression of the spring is found to be x_0. Determine (i) the common velocity of A and B at time t_0; and (ii) the spring constant. **[1984- 6 Marks]**

29. A bullet of mass M is fired with a velocity 50 m/s at an angle with the horizontal. At the highest point of its trajectory, it collides head-on with a bob of mass 3M suspended by a massless string of length 10/3 metres and gets embedded in the bob. After the collision, the string moves through an angle of 120°. Find

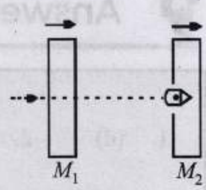
(i) the angle θ ;

(ii) the vertical and horizontal coordinates of the initial position of the bob with respect to the point of firing of the bullet. Take $g = 10\,m/s^2$

30. Three particles A, B and C of equal mass move with equal speed V along the medians of an equilateral triangle as shown in figure. They collide at the centroid G of the triangle. After the collision, A comes to rest, B retraces its path with the speed V. What is the velocity of C ? **[1982 - 2 Marks]**



31. A 20 gm bullet pierces through a plate of mass $M_1 = 1$ kg and then comes to rest inside a second plate of mass $M_2 =$ 2.98 kg. as shown. It is found that the two plates initially at rest, now move with equal velocities. Find the percentage loss in the initial velocity of the bullet when it is between M_1 and M_2. Neglect any loss of material of the plates due to the action of the bullet. **[1979]**



32. A body of mass m moving with velocity V in the X-direction collides with another body of mass M moving in Y-direction with velocity v. They coalesce into one body during collision. Calculate : **[1978]**

(i) the direction and magnitude of the momentum of the final body.

(ii) the fraction of initial kinetic energy transformed into heat during the collision in terms of the two masses.

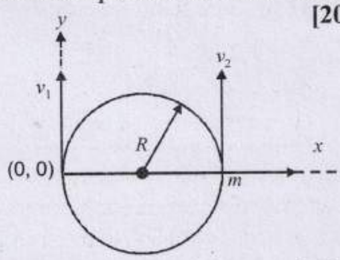
Topic-5: Miscellaneous (Mixed Concepts) Problems

3 Numeric Answer

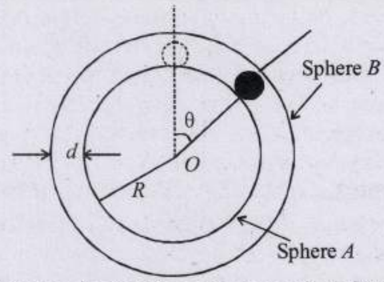
1. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass $m = 0.4\ kg$ is at rest on this surface. An impulse of 1.0 Ns is applied to the block at time $t = 0$ so that it starts moving along the x-axis with a velocity $v(t) = v_0 e^{-t/\tau}$, where v_0 is a constant and $\tau = 4s$. The displacement of the block, in *metres*, at $t = \tau$ is ________. Take $e^{-1} = 0.37$. **[Adv. 2017]**

10 Subjective Problems

2. A particle of mass m, moving in a circular path of radius R with a constant speed v_2 is located at point $(2R, 0)$ at time $t = 0$ and a man starts moving with a velocity v_1 along the +ve y-axis from origin at time $t = 0$. Calculate the linear momentum of the particle w.r.t. the man as a function of time. **[2003 - 2 Marks]**



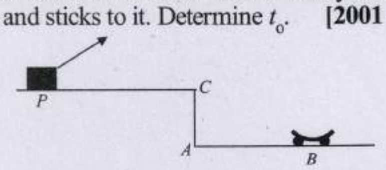
3. A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (see figure). The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter very slightly less than d. All surfaces are frictionless. The ball is given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is denoted by θ (shown in the figure). **[2002 - 5 Marks]**



(a) Express the total normal reaction force exerted by the sphere on the ball as a function of angle θ.

(b) Let N_A and N_B' denote the magnitudes of the normal reaction forces on the ball exerted by the sphere A and B, respectively. **Sketch** the variations of N_A and N_B as functions of $\cos\theta$ in the range $0 \le \theta \le \pi$ by drawing two **separate** graphs in your answer book, taking $\cos\theta$ on the horizontal axes.

4. A car P is moving with a uniform speed of $5\sqrt{3}$ m/s towards a carriage of mass 9 kg at rest kept on the rails at a point B as shown in figure. The height AC is 120 m. Cannon balls of 1 kg are fired from the car with an initial velocity 100 m/s at an angle 30° with the horizontal. The first cannon ball hits the stationary carriage after a time t_0 and sticks to it. Determine t_0. **[2001 - 10 Marks]**



At t_0, the second cannon ball is fired. Assume that the resistive force between the rails and the carriage is constant and ignore the vertical motion of the carriage throughout. If the second ball also hits and sticks to the carriage, what will be the horizontal velocity of the carriage just after the second impact?

Answer Key

Topic-1 : Work Done by Constant & Variable Force

1. (d)	2. (b)	3. (c)	4. (d)	5. (8)	6. (0.75)

Topic-2 : Energy

1. (c)	2. (b)	3. (b)	4. (d)	5. (b)	6. (c)	7. (5)	8. (4)	9. (a, d)	10. (a,b,d)
11. (a,b,d)	12. (d)	13. (c,d)	14. $A\rightarrow p, q, r, t; B\rightarrow q, s; C\rightarrow p, q, r, s; D\rightarrow p, r, t$						
15. (b)	16. (b)	17. (c)							

Topic-3 : Power

1. (b)	2. (b)	3. (5)

Topic-4 : Collisions

1. (b)	2. (a)	3. (d)	4. (d)	5. (c)	6. (d)	7. (4)	8. (5)
9. (30.00)	10. (a, c, d)	11. (a, d)	12. (a, b, d)	13. (a, c)	14. (a, d)	15. (b, d)	16. (a)
17. (c, d)	18. (b)	19. (b)	20. (c)	21. (d)			

Topic-5 : Miscellaneous (Mixed Concepts) Problems

1. (6.3)

Hints & Solutions

Topic-1: Work Done by Constant & Variable Force

1. **(d)** Radius of circular path = a

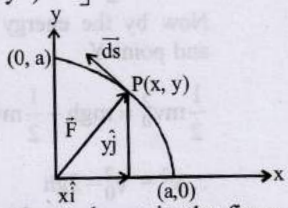
The equation of circle is $x^2 + y^2 = a^2$

Given : force

$$\vec{F} = K\left[\frac{x\hat{i}}{(x^2+y^2)^{3/2}} + \frac{y\hat{j}}{(x^2+y^2)^{3/2}}\right]$$

$$\vec{F} = K\left[\frac{x\hat{i}}{(a^2)^{3/2}} + \frac{y\hat{j}}{(a^2)^{3/2}}\right]$$

$$\vec{F} = \frac{K}{a^3}\left[x\hat{i} + y\hat{j}\right]$$



The force acts radially outwards as shown in the figure and the displacement is tangential to the circular path. Here the angle between the force which acts radially outwards and displacement which is tangential to the circular path is 90°

$\therefore$ Work done, $W = FS\cos\theta = 0$

2. **(b)** In a conservative field work done does not depend on the path *i.e.*, path independent. The gravitational field is a conservative field.

$\therefore \quad W_1 = W_2 = W_3$

3. **(c)** $dW = \vec{F}\cdot d\vec{S}$ and $\vec{F} = -K(y\hat{i} + x\hat{j})$ given

$d\vec{S} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

$dW = -K(y\,\hat{i} + x\,\hat{j}).(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k})$

$= -K(ydx + xdy)$

for (0, 0) to (a, 0)

$x \Rightarrow 0 \to a$

$y = 0$ and $dy = 0$

So, $dW = 0 \Rightarrow W = 0$

for (a, 0) to (a, a)

$y \Rightarrow 0$ to a

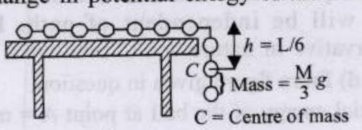
$x = a,\ dx = 0$

So, $dW = -Ka\,dy$

$$W = \int_0^a -Ka\,dy = -Ka^2$$

Hence total work done by the force on the particle, $W = -Ka^2$

4. **(d)** The work done in bringing the mass up will be equal to the change in potential energy of the mass.



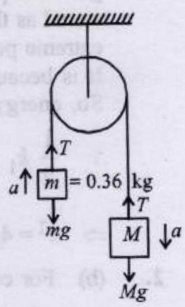
i.e., W = Change in potential energy

$$= mgh = \frac{M}{3}\times g \times \frac{L}{6} = \frac{MgL}{18}$$

5. **(8)** When the system is released,

$T - mg = ma$ ...(i)

$Mg - T = Ma$ ...(ii)



From eq. (i) & (ii)

$$a = \frac{(M-m)g}{M+m} = g/3$$

and $T = 4\,mg/3$

For block m = 0.36 kg

$u = 0,\ a = g/3,\quad t = 1,\ s = ?$

$$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\times\frac{g}{3}\times 1^2 = g/6 \quad (m = 0.72\text{ kg})$$

$\therefore$ Work done by the string on m

$$Ts\cos 0^\circ = 4\frac{mg}{3}\times\frac{g}{6}\times 1 = \frac{4\times 0.36\times 10\times 10}{3\times 6} = 8\text{ J}$$

6. **(0.75)** Given : Force, $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})$

and $\alpha = -1\text{ Nm}^{-1}$

We know that $dW = \vec{F}\cdot d\vec{r} = (\alpha y\,\hat{i} + 2\alpha x\,\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j})$

$\therefore dW = \alpha y dx + 2\alpha x dy$

Work done

From A → B dy = 0, as y = 1

$$\therefore W_1 = \int_0^1 \alpha y\,dx = \alpha\int_0^1 dx = \alpha$$

From B → C dx = 0, as x = 1

$$\therefore W_2 = \int_1^{0.5} 2\alpha n\,dy = \int_1^{0.5} 2\alpha dy = 2\alpha(-0.5) = -\alpha$$

From C → D dy = 0, as y = 0.5

$$\therefore W_3 = \int_1^{0.5} \alpha\times 0.5\,dx = -\frac{\alpha}{4}$$

From D → E dx = 0, as x = 0.5

$$\therefore W_4 = \int_{0.5}^{0} 2\alpha\times 0.5\,dy = -\frac{\alpha}{2}$$

From E → F dy = 0, as y = 0

$$\therefore W_5 = \int \alpha\times 0\times dx = 0$$

From F → A dx = 0 as x = 0

$$\therefore W_6 = \int 2\alpha\times 0\,dx = 0$$

$\therefore$ Total work done $W = W_1 + W_2 + W_3 + W_4 + W_5 + W_6$

$$= \alpha - \alpha - \frac{\alpha}{4} - \frac{\alpha}{2} = -\frac{3\alpha}{4} = \frac{-3(-1)}{4} = 0.75\text{J}$$

Topic-2: Energy

1. (c) Heres when the block B is displaced towards wall 1, only spring S_1 is compressed and S_2 is in its natural state as the other end of S_2 is free.

Therefore the energy stored in the system $= \frac{1}{2}k_1x^2$.

When the block is released, it will come back to the equilibrium position, gain momentum, overshoot to equilibrium position and move towards wall 2. As this happens, the spring S_1 comes to its natural length and S_2 gets compressed. The P.E. stored in the spring S_1 gets stored as the P.E. of spring S_2 when the block B reaches its extreme position after compressing S_2 by y.

It is because no friction anywhere.

So, energy is conserved

$\therefore \quad \frac{1}{2}k_1 x^2 = \frac{1}{2}k_2 y^2 \Rightarrow \frac{1}{2}\times kx^2 = \frac{1}{2}\times 4ky^2$

$\Rightarrow x^2 = 4y^2 \quad \therefore \frac{y}{x} = \frac{1}{2}$

2. (b) For conservative forces $\Delta U = -W$

$\Delta U = -\int_0^x Fdx$ or $\Delta U = -\int_0^x kxdx$

$\Rightarrow U_{(x)} - U_{(0)} = -\frac{kx^2}{2} \quad (\because U_{(0)} = 0)$

$\therefore \quad U_{(x)} = -\frac{kx^2}{2} \Rightarrow x^2 = \frac{-2U_x}{k}$

It represents a parabola below x – axis symmetrical

3. (b) Let x be the maximum extension of the string. Here mechanical energy is conserved, so decrease in the gravitational potential energy of spring mass system (Mgx)

= gain in spring elastic potential energy $\left(\frac{1}{2}kx^2\right)$

$$Mgx = \frac{1}{2}kx^2 \Rightarrow x = \frac{2Mg}{k}$$

4. (d)

5. (b) When spring is cut into pieces then the length of longer piece of spring $= \frac{2L}{3}$

Here the original length of spring be L and spring constant is K (given)

For a spring, $K \times L$ = constant $\quad [\because K = YA/l]$

Let K' be the spring cosntant of longer piece of spring

$\therefore \quad K \times L = \frac{2L}{3} \times K' \Rightarrow K' = \frac{3}{2}K$

6. (c) K.E. $= \frac{p^2}{2m}$; [K.E. = Kinetic energy; P = momentum]

$KE_1 = KE_2 \quad \therefore \frac{p_1^2}{m_1} = \frac{p_2^2}{m_2}$

$\therefore \quad \frac{p_1^2}{p_2^2} = \frac{m_1}{m_2} \Rightarrow \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

7. (5) Work done = Increase in P.E. + gain in K.E.

$F \times d = mgh$ + gain in K.E.

$18 \times 5 = 1 \times 10 \times 4$ + gain in K.E.

$\therefore$ Gain in K.E. = 50 J = 10n $\therefore n = 5$

8. (4) Here, loss in K.E. of the block

= gain in P.E. of the spring + work done against friction

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \mu mg.x \Rightarrow v = \sqrt{\frac{2\mu mgx + kx^2}{m}}$.

$v = \sqrt{\frac{2\times 0.1\times 0.18\times 10\times 0.6 + 2\times 0.6\times 0.6}{0.18}}$

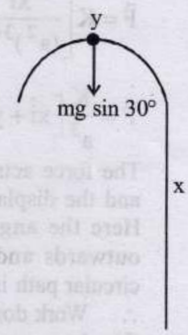
$\therefore v = \frac{4}{10} = \frac{N}{10} \quad \therefore N = 4$

9. (a, d) At point Y the centripetal force provided by the component of weight mg

$\therefore mg \sin 30° = \frac{mv^2}{R}$

$\therefore v^2 = \frac{gR}{2}$ ...(ii)

Now by the energy conservation between bottom point and point Y



$\frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv^2$

$\therefore v^2 = v_0^2 - 2gh$ ...(i)

$\therefore$ From eq. (i)

$\frac{gR}{2} = v_0^2 - 2gh$

Hence option (a) is correct.

At point x and z of circula path, the points are at same height but less then h. So the velocity more than at point y.

So required centripetal force $= \frac{mv^2}{r}$ is maximum at points x and y.

10. (a, b, d) $G\frac{dk}{dt} = \gamma t$ and $k = \frac{1}{2}mV^2 \quad \therefore \frac{d}{dt}\left(\frac{1}{2}mV^2\right) = \gamma t$

$\Rightarrow \quad \frac{m}{2}\times 2V\frac{dV}{dt} = \gamma t \quad \therefore mV\frac{dV}{dt} = \gamma t$

$\therefore \quad m\int_0^V VdV = \gamma\int_0^t tdt \quad \Rightarrow \frac{mV^2}{2} = \frac{\gamma t^2}{2}$

$\therefore V = \sqrt{\frac{\gamma}{m}}\times t$. i.e., $V \propto t$; $V = \frac{ds}{dt} = \sqrt{\frac{\gamma}{m}}t \Rightarrow s = \sqrt{\frac{\gamma}{m}}\frac{t^2}{2}$

So V is proportional to 't' and distance cannot be proportional to 't'.

Now $F = ma = m\frac{dV}{dt} = m\frac{d}{dt}\left[\sqrt{\frac{\gamma}{m}}\times t\right] = m\sqrt{\frac{\gamma}{m}} = \sqrt{\gamma m}$ = constant

Since force applied is constant and displacement between any two points on x-axis will also be constant, thus work done will be independent of path. Hence force is conservative in nature.

11. (a, b, d) From figure given in question,

Potential energy of the ball at point A = mgh_A

Potential energy of the ball at point B = 0

Potential energy of the ball at point C = mgh_C

Total energy at point A, $E_A = K_A + mgh_A$

Total energy at point B, $E_B = K_B$

Total energy at point C, $E_C = K_C + mgh_C$

As body rolls between A and B and between B and C there is no friction. So energy should be conserved here

By law of conservation of energy.

$E_A = E_B = E_C$

As $E_A = E_C$

$K_A + mgh_A = K_C + mgh_C$

So, If $h_A > h_C \Rightarrow K_A < K_C$. So option (b) is correct

If $h_A < h_C \Rightarrow K_A > K_C$

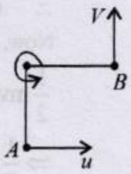
Doesn't matter if $h_A > h_C$ or $h_A < h_C$, we will always have $K_B > K_C$ because $E_A = E_B = E_C$. So option (a) and (d) is also correct.

12. **(d)** From principle of conservation of energy

$(K.E.)_B + (P.E.)_B = (K.E.)_A + (P.E.)_A$,

or, $\frac{1}{2}mv^2 + mgL = \frac{1}{2}mu^2 + 0$

$\Rightarrow \quad v = \sqrt{u^2 - 2gL}$ ... (i)



Change in velocity $|\Delta \vec{v}|$

$= |\vec{v} - \vec{u}| = \sqrt{v^2 + u^2}$

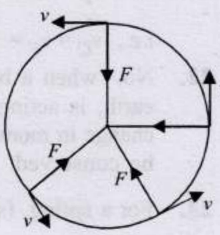
Putting the value of v from eq. (i)

$|\Delta \vec{v}| = \sqrt{2(u^2 - g\ell)}$

13. **(c, d)** As particle is acted upon by a force of constant magnitude and is perpendicular to the velocity of the particle so it is a case of uniform circular motion.

The force is constant in magnitude, this show the speed is not changing and hence kinetic energy will remain constant.

The velocity and acceleration changes continuously due to change in the direction.



14. **A → p, q, r, t; B → q, s; C → p, q, r, s; D → p, r, t**

For A

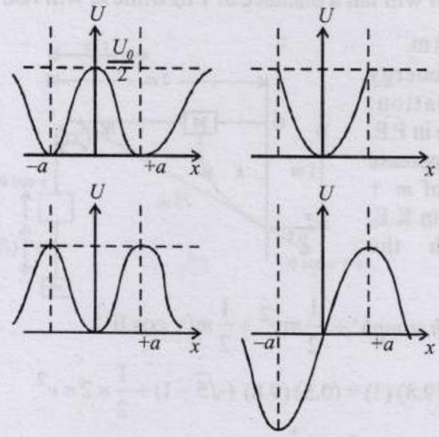
$$Fx = -\frac{dU}{dx} = \frac{-d}{dx}\frac{U_0}{2}\left[1 - \left(\frac{x}{a}\right)^2\right]^2 = \frac{-2U_0}{a^4}(x-a)x(x+a)$$

$F = 0$ at $x = 0, x = a, x = -a$

and $U = 0$ at $x = -a, x = a$

For attractive force, $F \alpha - x$. So (s) does not apply.

At $x = -a$, $\frac{d^2U}{dx^2}$ is +ve, which implies curve is having minima at this point and will oscillate about this point as it is minimum P.E. and total energy $\left(\frac{U_0}{4}\right)$ of particle is less than maximum P.E. $\left(\frac{U_0}{2}\right)$



For B $F_x = \frac{-dU}{dx} = -U_0\left(\frac{x}{a^2}\right)$

For C $F_x = \frac{-dU}{dx} = U_0\frac{e^{-x^2/a^2}}{a^3}x(x-a)(x+a)$

For D $F_x = -\frac{dU}{dx} = \frac{U_0}{2a^3}[(x-a)(x+a)]$

15. **(b)** As $W_{\text{all forces}} = \Delta K \Rightarrow W_{mg} + W_{fr} = \frac{1}{2}mv^2 - 0$

$\Rightarrow mgh - 150 = \frac{1}{2}mv^2$

$\Rightarrow mgR \sin 30° - 150 = \frac{1}{2}mv^2 \Rightarrow 1 \times 10 \times 40 \times \frac{1}{2} - 150 = \frac{v^2}{2}$

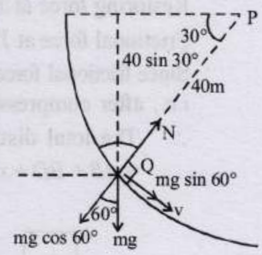
$\Rightarrow v = 10$ m/s

$N - mg \cos \theta$ will provide required centripetal force

$N - mg \cos \theta = \frac{mv^2}{R}$

$N = mg \cos \theta + \frac{mv^2}{R}$

$= 1 \times 10 \times \frac{1}{2} + \frac{1 \times (10)^2}{40} = 7.5$ N



16. **(b)** As discussed earlier, we get $v = 10$ m/s.

17. **(c)** In the first case the mechanical energy is completely converted into heat becuase of friction. *i.e.*, Decrease in mechanical energy = $\frac{1}{2}mv^2$.

While is second case, a part of mechanical energy is converted into heat due to fiction but another part of mechanical energy is retained in the form of potential energy of the block. *i.e.*,

Decrease in mechanical energy = $\frac{1}{2}mv^2 - mgh$

Therefore statement 1 is correct.

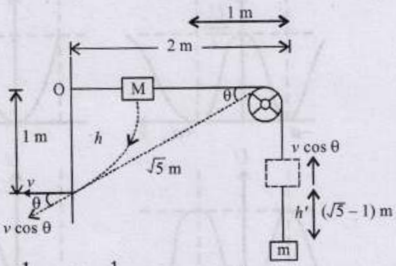
Statement 2 is wrong. The coefficient of friction between the block and the surface does not depend on the angle of inclination.

18. Let point mass hit the wall with the speed v.

Then, velocity of mass m at this instant = $v \cos \theta = \frac{2}{\sqrt{5}}v$.

Further M will fall a distance of 1 m while m will rise up by $(\sqrt{5}-1)$ m.

From energy conservation, Decrease in P.E. of M = increase in P.E. of m + increase in K.E. of both the blocks.



$$Mgh = mgh' + \frac{1}{2}mv^2 + \frac{1}{2}m(v\cos\theta)^2$$

or, $(2)(9.8)(1) = (0.5)(9.8)(\sqrt{5}-1) + \frac{1}{2}\times 2\times v^2$

$$+\frac{1}{2}\times 0.5\times\left(\frac{2v}{\sqrt{5}}\right)^2$$

Solving we get, $v = 3.29$ m/s.

19. K.E. of block = work against friction + P.E. of spring

$$\frac{1}{2}mv^2 = \mu_k mg\,(BD+x) + \frac{1}{2}kx^2$$

$$\frac{1}{2}mv^2 = \mu_k mg\,(2.14+x) + \frac{1}{2}kx^2$$

$$\frac{1}{2}\times 0.5\times 3^2 = 0.2\times 0.5\times 9.8\,(2.14+x) + \frac{1}{2}\;2\times x^2$$

$$2.14 + x + x^2 = 2.25 \therefore x^2 + x - 0.11 = 0$$

On solving, we get $x = \frac{1}{10} = 0.1$

The spring gets compressed by 0.1 m

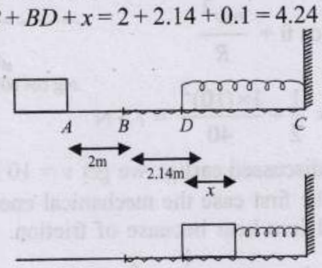
Restoring force at $Y = kx = 2\times 0.1 = 0.2\,N$

Frictional force at $Y = \mu_s mg = 0.22\times 0.5\times 9.8 = 1.078\,N$

Since frictional force > restoring force, the body will stop here *i.e.*, after compressing the spring by x

$\therefore$ The total distance travelled

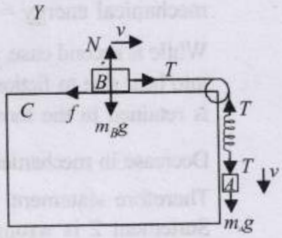
$= AB + BD + x = 2 + 2.14 + 0.1 = 4.24$ m.



20. Here the net force acting on A and B is zero. Since the blocks A and B are moving with constant velocity.

Let the extension of the spring be x.

There will be no friction force between block A and C



$\because f = \mu N$. Here there is no normal reaction on A because A is not pushing C so frictional force between block A and C, $f = \mu N = 0$

Applying $F_{net} = ma$ on A, $m_A g - T = m_A\times 0$

$\therefore T = m_A g$ ... (i)

Applying $F_{net} = ma$ on B,

$T - f = m_B\times 0$

$\therefore T = f = \mu N$

$= \mu m_B g$ ... (ii)

From (i) and (ii)

$$\mu m_B g = m_A g \Rightarrow m_B = \frac{m_A}{\mu} = \frac{2}{0.2} = 10\,\text{kg}$$

Here the force acting on the spring is the tension equal to restoring force

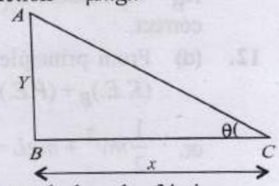
$\therefore T = kx \Rightarrow x = \frac{T}{k}$ $\therefore x = \frac{m_A g}{k} = \frac{19.6}{k}$

Energy stored in spring

$$U = \frac{1}{2}kx^2 = \frac{1}{2}\times k\times\frac{19.6}{k}\times\frac{19.6}{k} = \frac{19.6\times 19.6}{2\times 1960} = 0.098\,\text{J}$$

21. Here, K.E. at C = Loss in P.E. – work done by friction

In both cases work done by friction = μmgx



or, $\frac{1}{2}mv_c^2 = mg\,y - \mu mgx$

$= gy - \mu gx$

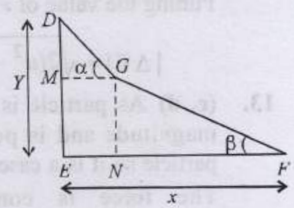
$\therefore v_c = \sqrt{2g\,(y-\mu x)}$

Now, K.E. at F = loss in P.E. – work done by friction

$$\frac{1}{2}mv_f^2 = mgy - (\mu\,mg\cos\alpha)DG - (\mu\,mg\cos\beta)GF$$

$$\Rightarrow \frac{1}{2}mv_f^2 = mgy - \mu\,mg\,(DG\cos\alpha + GF\cos\beta)$$

$$\Rightarrow \frac{1}{2}mv_F^2 = mgy - \mu mgx$$



$$\Rightarrow \frac{1}{2}v_F^2 = gy - \mu gx$$

$\therefore v_F = \sqrt{2g\,(y-\mu x)}$

i.e., $v_C = v_F = \sqrt{2gy - 2\mu gx}$

22. No. when a ball is thrown up the gravitational pull of earth, is acting on the ball which is responsible for the change in momentum. As $F_{net}\neq 0$, so momentum will not be conserved.

23. For a spring, (spring constant) $k\times$ (length) l = Constant

If length is made one third, *i.e.*, $\frac{l}{3}$ the spring constant becomes three times *i.e.*, 3K.

24. Using Kinetic energy (K.E.) and momentum relation,

K.E. = $\frac{p^2}{2m}$ For equal value of p, K.E. $\propto \frac{1}{\text{mass}}$

Topic-3: Power

1. **(b)** The centripetal acceleration $a_c = k^2 r t^2$

or, $\frac{v^2}{r} = k^2 r t^2$ $\therefore V = krt$

Now, $a_t = \frac{dv}{dt} = kr$

Power, $P = F_t v = m a_t v$

$\therefore$ Power $= mk^2r^2t$

2. (b) If machine is lubricated with oil friction is reduced.

Mechanical efficiency $= \dfrac{\text{Output work}}{\text{Input energy}}$

Due to less friction output work will increase. Thus the mechanical efficiency increases.

Mechanical advantage, M.A. $= \dfrac{\text{load}}{\text{effort}}$

3. (5) Using, work – energy theorem, Δ K.E. = W = P × t

$$\frac{1}{2}mv^2 = P \times t \therefore v = \sqrt{\frac{2Pt}{m}} = \sqrt{\frac{2 \times 0.5 \times 5}{0.2}} = 5ms^{-1}$$

Topic-4: Collisions

1. (b) As tennis ball is dropped, so initial velocity $u = 0$

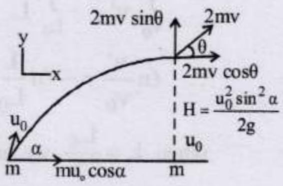
$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}m[u+at]^2 = \frac{1}{2}m[0+gt]^2$$

$\therefore K.E = \frac{1}{2}mg^2t^2 \qquad \therefore K.E \propto t^2$ ...(i)

i.e., The relation between k and t is parabolic.

First the kinetic energy will increase as per eq (i). As the balls touches the ground it starts deforming and loses its K.E., when the deformation is maximum, K.E. = 0. As the ball moves up it loses K.E. and gain gravitational potential energy in the same time interval. These characteristics are best illustrated by kt graph shown in (b).

2. (a) Height, $h = \dfrac{u_0^2 \sin^2 \alpha}{2g}$



using $v^2 - u^2 = 2gh$

$$v_1^2 - u_0^2 = 2(-g)\left[\frac{u_0^2 \sin^2 \alpha}{2g}\right]$$

$\Rightarrow \quad v_1^2 = u_0^2(1 - \sin^2 \alpha) = u_0^2 \cos^2 \alpha$

$\Rightarrow \quad v_1 = u_0 \cos \alpha$

Applying conservation of linear momentum in Y-direction

$2mv \sin \theta = mv_1 = mu_0 \cos\alpha$ ...(i)

Applying conservation of linear momentum in X-direction

$2mv \cos\theta = mu_0 \cos\alpha$ ...(ii)

Dividing (i) and (ii) we get

$\tan \theta = 1 \qquad \therefore \theta = 45° = \dfrac{\pi}{4}$

3. (d) Maximum energy loss $= \dfrac{P^2}{2m} - \dfrac{P^2}{2(m+M)}$

$$\left[\because K.E. = \frac{P^2}{2m} = \frac{1}{2}mv^2\right]$$

$$= \frac{P^2}{2m}\left[\frac{M}{(m+M)}\right] = \frac{1}{2}mv^2\left\{\frac{M}{m+M}\right\}$$

Statement II is a case of perfectly inelastic collision.

By comparing the equation given in statement I with above equation, we get

$$f = \left(\frac{M}{m+M}\right) \text{ instead of } \left(\frac{m}{M+m}\right)$$

Hence statement I is wrong and statement II is correct.

4. (d) Let after 't' time both ball and bullet hit the ground.

Then, $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 5}{10}} = 1$ sec.

After collision let V_{ball} be velocity of ball and V_{bullet} be velocity of bullet.

So, $20 = V_{ball} \times 1 \Rightarrow V_{ball} = 20$ m/s

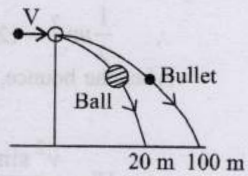
$100 = V_{bullet} \times 1 \Rightarrow V_{bullet} = 100$ m/s

By law of conservation of momentum

$0.01\,V = 0.01 \times 100 + 0.2 \times 20$

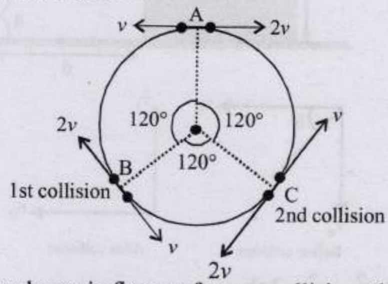
$\Rightarrow 0.01\,V = 1 + 4$

$\Rightarrow V = \dfrac{5}{0.01} = 500$ m/s.



5. (c) According to question, between collision, the particles move with constant speed.

At first collision one particle having speed $2v$ will rotate 2 × 120° = 240° while other particle having speed v will rotate 120°. Hence, first collision takes place at B. At first collision, they will exchange their velocities as the collision is elastic and the particles have equal masses. Again second collision takes place at C.



Now, as shown in figure, after two collisions they will again reach at point A.

6. (d) $F_{ext} = (m_1 + m_2)g$ and $\Delta t = 2t_0$

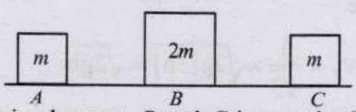
$$F_{ext} = \frac{\Delta p}{\Delta t}$$

$$\therefore \quad \Delta p = F_{ext}\,\Delta t = (m_1\vec{v}_1{}' + m_2\vec{v}_2{}') - (m_1\vec{v}_1 + m_2\vec{v}_2)$$

$$= (m_1 + m_2)\,g \times 2t_0$$

7. (4) Hence, Just after collision with A velocity of B

$$v_B = \frac{(m_B - m_A)u_B}{m_B + m_A} + \frac{2m_A u_A}{m_A + m_B} = \frac{0 + 2m \times 9}{m + 2m} = 6\text{m/s}$$



The collision between B and C is completely inelastic

$\therefore \quad m_B v_B = (m_B + m_c)\,v$

$\Rightarrow \quad v = \dfrac{6 \times 2m}{2m + m} = 4$m/s.

8. (5) Due to elastic head on collision of equal mass m of bob, velocity at the highest point of bob tied to string ℓ_1 is acquired by the bob tied to string ℓ_2.

$$\therefore \sqrt{g\ell_1} = \sqrt{5g\ell_2} \quad \Rightarrow \quad \frac{\ell_1}{\ell_2} = 5$$

9. **(30.00)** Maximum height,

$$H = \frac{u^2 \sin^2\theta}{2g} \Rightarrow 120 = \frac{u^2\left(\frac{1}{2}\right)}{2g}$$

$\therefore \quad u^2 = 480\text{ g}$

Upon hitting the ground, it loses half of its kinetic energy

$$\therefore \quad K.E_{initial} = \frac{1}{2}mu^2 = 240\text{ mg}$$

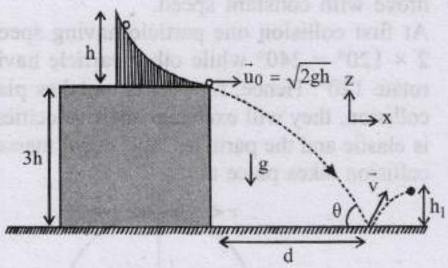
$$K.E_{final} = \frac{1}{2}(240\text{ mg}) = 120\text{ mg}$$

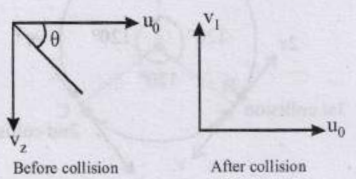
$$\therefore \quad \frac{1}{2}mv^2 = 120\text{ mg} \qquad \therefore v^2 = 240\text{ g}$$

After the bounce, the maximum height the ball reaches

$$\therefore \quad H' = \frac{v^2\sin^2\theta}{2g} = \frac{240g \times \left(\frac{1}{4}\right)}{2g} = 30\text{ m}$$

10. **(a, c, d)**





From $v^2 - u^2 = 2gh$

$0^2 - u_0^2 = 2gh \;\therefore u_0 = \sqrt{2gh} \;\therefore \vec{u_0} = \sqrt{2gh}\,\hat{x}$

and $v_z = \sqrt{2g(3h)}$

$$\therefore \tan\theta = \frac{v_z}{u_0} = \frac{\sqrt{2g(3h)}}{\sqrt{2gh}} = \sqrt{3} \;\therefore \theta = 60°$$

$$\text{Distance } d = u_0 t = \sqrt{2gh} \times \sqrt{\frac{2\times 3h}{g}} = 2\sqrt{3}h$$

After collision only velocity along z–direction will change

$$v_1 = ev_z = \frac{1}{\sqrt{3}} \times \sqrt{2g(3h)} = \sqrt{2gh}$$

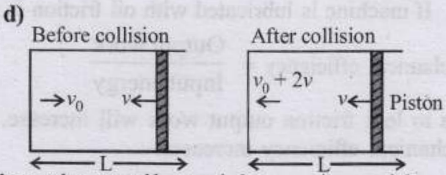
$$\therefore \vec{v} = v_1\hat{k} + u_0\hat{i} = \sqrt{2gh}\,\hat{k} + \sqrt{2gh}\,\hat{i} = \sqrt{2gh}\left[\hat{i} + \hat{k}\right]$$

$$\text{Height } h_1 = \frac{v_1^2}{2g} = \frac{\left(\sqrt{2gh}\right)^2}{2g} = h$$

$$\therefore d/h_1 = \frac{2\sqrt{3}h}{h} = 2\sqrt{3}$$

Therefore options (a, c, d) are correct.

11. **(a, d)**



When the small particle moving with velocity v_0 undergoes an elastic collision with the heavy movable piston moving with velocity v, it acquires a new velocity $v_0 + 2v$. So, the increase in velocity after every collision is $2v$.

Time period of collision when the piston is at a distance 'L' from the closed end

$$T = \frac{\text{distance}}{\text{speed}} = \frac{2L}{v'}$$

Where v' is the speed of the particle at that time.

$\therefore$ Frequency or rate at which the particle strikes the piston

$$= \frac{v'}{2L}$$

The rate of change of speed of the particle

$$= \frac{dv'}{dt} = (\text{frequency}) \times 2v \;\therefore\; dv' = \frac{v'}{2L}2v\,dt$$

$$\therefore \frac{dv'}{v'} = \frac{v\,dt}{L} = \frac{-dL}{L}$$

Where dL is the distance travelled by the piston in time dt. The minus sign indicates decrease in 'L' with time.

$$\therefore \int_{V_0}^{V}\frac{dv'}{v'} = -\int_{L_0}^{x}\frac{dL}{L}$$

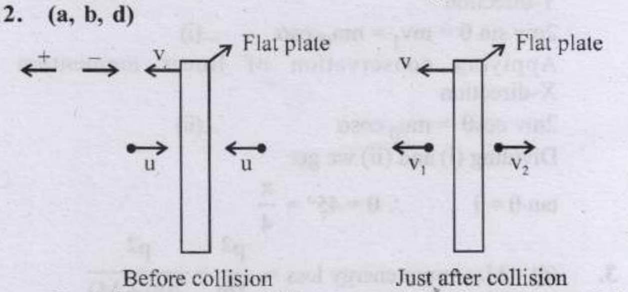
$$\therefore \ell n\frac{v'}{v_0} = -\ell n\frac{L}{L_0} \qquad \text{or} \qquad |v'| = \frac{v_0L_0}{L}$$

$$\text{when } L = \frac{L_0}{2} \text{ we have } |V'| = \frac{v_0L_0}{L_0/2} = 2V_0$$

$$\therefore K.E_{L_0/2} = \frac{1}{2}m(2v_0)^2$$

$$\therefore K.E_{L_0} = \frac{1}{2}mv_0^2 \qquad \therefore \frac{K.E_{L_0/2}}{K.E_{L_0}} = 4$$

12. **(a, b, d)**



For left gaseous particle **For right gaseous particle**

$$1 = \frac{v_1 - v}{v + u} \qquad\qquad 1 = \frac{v + v_2}{u - v}$$

$\therefore v_1 = u + 2v \qquad\qquad \therefore v_2 = u - 2v$

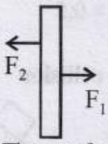
$\therefore \Delta v_1 = 2u + 2v \qquad\qquad$ and $\Delta v_2 = 2u - 2v$

$$\text{Now } F_1 = \frac{dp_1}{dt} = \rho A(u+v)(2u+2v)$$

$$\text{and } F_2 = \frac{dp_2}{dt} = \rho A(u-v)(2u-2v)$$

$$\therefore F_1 = 2\rho A(u+v)^2 \quad \text{and } F_2 = 2\rho A(u-v)^2$$

ΔF is the net force due to the air molecules on the plate.



$\Delta F = F_1 - F_2 = 8\rho Auv \therefore P = \frac{\Delta F}{A} = 8\rho uv$

The net force $F_{net} = F - \Delta F = ma$

$\therefore F - 8\rho Auv = ma$

Due to viscosity, plate will eventually reach terminal velocity. So now plate will move with constant velocity.

13. **(a, c)** According to law of conservation of linear momentum

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$1 \times u_1 + 5 \times 0 = 1(-2) + 5(v_2)$

$\Rightarrow u_1 = -2 + 5v_2$ ...(i)

The coefficient of restitution

$e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow 1 = \frac{v_2 - (-2)}{u_1 - 0} \Rightarrow u_1 = v_2 + 2$ ...(ii)

From eq (i) & (ii) $u_1 = 3$ m/s and $v_2 = 1$ m/s

Hence total momentum of the system = 3 kg m/s and K.E.cm = 0.75 J

14. **(a, d)** From law of conservation of linear momentum

The initial linear momentum of the system, $p\hat{i} - p\hat{i} = 0$

$\therefore$ Final linear momentum should also be zero *i.e.*, $p_1' + p_2' = 0$

Option a :

$p_1' + p_2' = (a_1 + a_2)\hat{i} + (b_1 + b_2)\hat{j} + c_1\hat{k}$ = Final momentum.

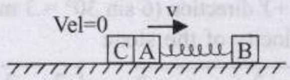
It is given that a_1, b_1, c_1, a_2, b_2 and c_2 have non-zero values. If $a_1 = x$ and $a_2 = -x$. Also if $b_1 = y$ and $b_2 = -y$ then the $\hat{i}$ and $\hat{j}$ components become zero. But the third term having $\hat{k}$ component is non-zero. This gives a definite final momentum to the system which violates conservation of linear momentum, so this is a wrong option.

Option d:

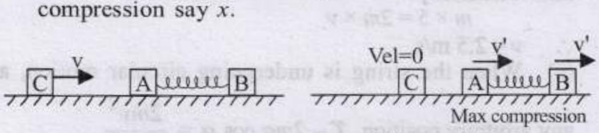
$p_1' + p_2' = (a_1 + a_2)\hat{i} + 2b_1\hat{j} \neq 0$ because $b_1 \neq 0$

Following the same reasoning as above the option d is also wrong.

15. **(b, d)** Just after the collision of *C* with *A*, *C* stops and *A* acquires a velocity *v* because of head-on elastic collision between identical masses.



When *A* starts moving towards right, the spring suffer a compression due to which *B* also starts moving towards right. The compression of the spring continues till there is relative velocity between *A* and *B*. When this relative velocity becomes zero, both *A* and *B* move with the same velocity *v'* and the spring is in a state of maximum compression say *x*.



From principle of conservation of linear momentum

$mv = mv' + mv' \Rightarrow v' = \frac{v}{2}$

$\therefore$ K.E. of $A - B$ system at maximum compression,

$\frac{1}{2}mv'^2 + \frac{1}{2}mv'^2 = mv'^2 = \frac{mv^2}{4} \quad \left(\because v' = \frac{v}{2}\right)$

Applying energy conservation

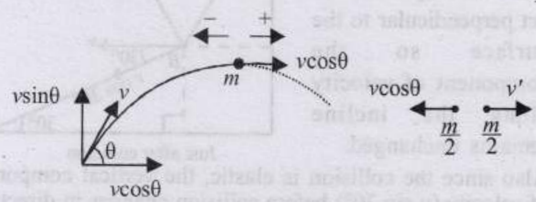
$\frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + \frac{1}{2}mv'^2 + \frac{1}{2}Kx^2$

$\Rightarrow \frac{1}{2}mv^2 = \frac{1}{4}mv^2 + \frac{1}{2}Kx^2$

$\Rightarrow \frac{1}{2}Kx^2 = \frac{1}{4}mv^2 \quad \therefore \quad x = v\sqrt{\frac{m}{2K}}$

16. **(a)** Let *v'* be the speed of other piece of shell after the collision.

As one piece retraces its path, the speed of this piece just after explosion should be $v \cos\theta$



Applying conservation of linear momentum at the highest point

$m(v\cos\theta) = \frac{m}{2} \times v' - \frac{m}{2} \times v\cos\theta$

$\therefore \quad v' = 3v\cos\theta$

17. **(c, d)** In inelastic collision only momentum of the system may remain conserved.

(a) is incorrect because the momentum of ball changes in magnitude as well as direction.

(b) is incorrect because on collision, some mechanical energy is converted into heat, sound energy.

(c) is correct because for earth + ball system the impact force is an internal force.

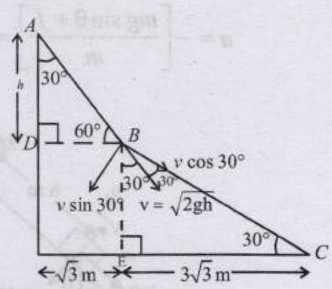
(d) is correct. Total energy of the ball and the earth is conserved.

18. **(b)** As the inclined plane is frictionless,

The K. E . at B = P.E. at *A*

$\frac{1}{2}mv^2 = mgh$

$\Rightarrow v = \sqrt{2gh}$



In ΔADB, $\tan 60° = \frac{h}{\sqrt{3}}$

$\therefore h = 3$ m

$\therefore v = \sqrt{6g} = \sqrt{60}$ m/s

This is the velocity of the block just before collision. This velocity makes an angle of 30° with the vertical. Also in right angled triangle *BEC*, $\angle EBC = 60°$. Therefore v makes an angle of 30° with the second inclined plane *BC*. The component of v along *BC* is $v \cos 30°$.

It is given that the collision at *B* is perfectly inelastic therefore the impact forces act normal to the plane such that the vertical component of velocity becomes zero. The component of velocity along the incline *BC* remains unchanged and is equal to $v \cos 30°$

$$= \sqrt{60} \times \frac{\sqrt{3}}{2} = \sqrt{\frac{180}{4}} = \sqrt{45}\,\text{m/s}$$

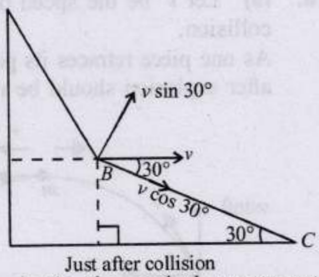
19. (b) In ΔBCE, $\tan 30° = \frac{BE}{CE} \Rightarrow \frac{1}{\sqrt{3}} = \frac{BE}{3\sqrt{3}} \Rightarrow BE = 3\text{m}$

From mechanical energy conservation principle,
Mechanical energy at B = mechanical energy at C

$$\frac{1}{2}M\left(\sqrt{45}\right)^2 + M \times 10 \times 3 = \frac{1}{2}Mv_c^2$$

$$45 + 60 = v_c^2 \qquad \therefore v_c = \sqrt{105}\,\text{m/s}$$

20. (c) The velocity of the block along BC just before collision is v cos 30°. The impact forces act perpendicular to the surface so the component of velocity along the incline remains unchanged.



Just after collision

Also since the collision is elastic, the vertical component of velocity (v sin 30°) before collision changes in direction, the magnitude remaining the same as shown in the figure. So the rectangular components of velocity after collision are as shown in the figure. This means that the final velocity of the block should be horizontal making an angle 30° with BC. Therefore the vertical component of the final velocity of the block is zero.

21. (d) Statement 1 : For an elastic collision, the coefficient of restitution = 1

$$e = \frac{|v_2 - v_1|}{|u_1 - u_2|} \Rightarrow |v_2 - v_1| = |u_1 - u_2|$$

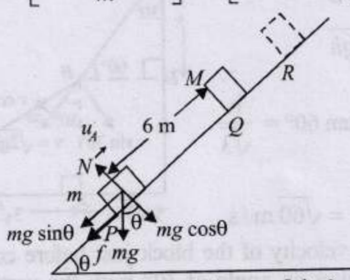
$\Rightarrow$ Relative velocity after collision is equal to relative velocity before collision. But in the statement relative speed is given.

Statement 2 : Linear momentum remains conserved in an elastic collision. This statement is true.

22. From *P* to *Q*.

Given : $u = 10$ m/s and $\mu = 0.25$; $s = 6$ m

$$a = -\left[\frac{mg\sin\theta + f}{m}\right] = -\left[\frac{mg\sin\theta + \mu mg\cos\theta}{m}\right]$$



$$= -[g\sin\theta + \mu g\cos\theta] = -g[\sin\theta + \mu\cos\theta]$$
$$= -10\,[0.05 + 0.25 \times 0.99] = -2.99\text{ m/s}^2$$

Using, $v^2 - u^2 = 2as \Rightarrow v^2 = 100 + 2(-2.99) \times 6 \Rightarrow v = 8$ m/s

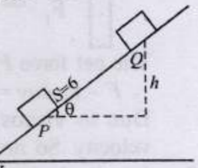
Hence velocity of mass m just before collision = 8 m/s.
The velocity of mass M just before collision = 0 m/s (given).
Let v_1 be the velocity of mass m after collision and v_2 be the velocity of mass M after collision. **Body of mass *M* moving from *Q* to *R* and coming to rest.** After collision,

$$u = v_2 \Rightarrow v = 0$$
$$a = -2.99\text{ m/s}^2 \Rightarrow s = 0.5$$

$$v^2 - u^2 = 2as \Rightarrow (0)^2 - v_2^2 = 2(-2.99) \times 0.5$$

$\Rightarrow \quad v_2 = 1.73$ m/s

Body of mass *m* moving from *Q* to *P* after collision



$$\sin\theta = \frac{h}{6}$$

$$h = 6\sin\theta = 6\sin 60° = 6 \times 0.05$$

$$u = v_1$$
$$v = +1\text{m/s}$$

$$(\text{K.E.} + \text{P.E.})_{\text{initial}} = (\text{K.E.} + \text{P.E.})_{\text{final}} + W_{\text{friction}}$$

$$\frac{1}{2}mv_1^2 + mgh = \frac{1}{2}mv^2 + 0 + \mu mgs$$

$$\frac{1}{2}v_1^2 + 10 \times (6 \times 0.05) = \frac{1}{2}(1)^2 + 0.25 \times 10 \times 6$$

$$v_1 = -5\text{ m/s}$$

$\therefore$ **Coefficient of restitution**

$$e = \left|\frac{\text{relative velocity of separation}}{\text{relative velocity of approach}}\right|$$

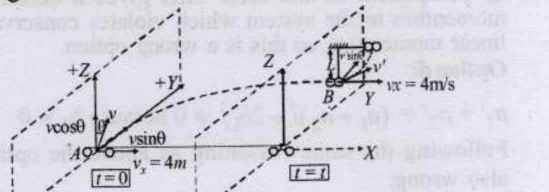
$$\left|\frac{-5 - 1.73}{8 - 0}\right| = 0.84$$

Applying conservation of linear momentum before and after collision,

$$mv + M \times 0 = m \times v_1 + m \times v_2$$
$$2 \times 8 + M \times 0 = 2 \times (-5) + M(1.73)$$

$$\therefore \quad M = \frac{26}{1.73} = 15.02\,\text{kg}$$

23. When the stone reaches the point B, the component of velocity in the $+Z$ direction (v cos θ) becomes zero due to the gravitational force in the $-Z$ direction.



The stone has two velocities at B
v_x in the $+X$ direction (4 m/s)
v sin θ in the $+Y$ direction (6 sin 30° = 3 m/s)
Resultant velocity of the stone

$$v' = \sqrt{(v_x)^2 + (v\sin\theta)^2} = \sqrt{4^2 + 3^2} = 5\,\text{m/s}$$

(i) Applying conservation of linear momentum at B for collision with a mass of equal magnitude.
Since, the collision is completely inelastic the two masses will stick together. v is the velocity of the two masses just after collision.]

$$m \times 5 = 2m \times v$$

$\therefore \quad v = 2.5$ m/s

(ii) When the string is undergoing circular motion, at any arbitrary position, $T - 2mg\cos\alpha = \frac{2mv^2}{\ell}$

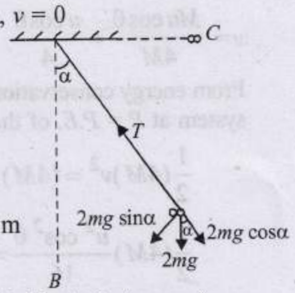
According to question, $T = 0$ when $\alpha = 90°$

$$\therefore \quad 0 - 0 = \frac{2mv^2}{\ell} \Rightarrow v = 0$$

i.e., in the horizontal position, $v = 0$
Applying energy conservation from B to C,

$$\frac{1}{2}2mv^2 = 2mg\ell$$

$$\Rightarrow \ell = \frac{v^2}{2g} = \frac{(2.5)^2}{2\times 9.8} = 0.318 \text{ m}$$



24. Object of mass m collides with block B of mass $4m$. Since the collision is elastic in nature applying conservation of linear momentum

$mv = (4\text{m})\,u + mv'$

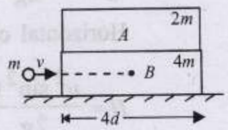
where u is the velocity of mass 4m after collision and v' is the velocity of mass m

$\Rightarrow \quad v' = v - 4u \quad$ ... (i)

Applying conservation of kinetic energy

Also, $\frac{1}{2}mv^2 = \frac{1}{2}(4\text{m})u^2 + \frac{1}{2}mv'^2$

$\Rightarrow \quad v^2 = 4u^2 + v'^2 \quad$ ... (ii)



From eq. (i) & (ii)

$$v^2 = 4u^2 + (v-4u)^2 \Rightarrow u = \frac{2v}{5}$$

block B starts moving but the block A remains at rest. As there is no friction between A and B

For block A to topple, block B should move a distance $2d$.

Let the retardation produced in B due to friction force between B and the table be a

$$F = \mu N \Rightarrow (4\,m)a = \mu\,(6\,mg) \Rightarrow a = 1.5\,\mu g$$

For the motion of B,

$$u = \frac{2v}{5},\ v = 0,\ s = 2\text{d},\ a = -1.5\,\mu g$$

Now, $v^2 - u^2 = 2as \Rightarrow (0)^2 - \left(\frac{2v}{5}\right)^2 = 2(-1.5\mu g)2d$

$$\Rightarrow \quad v = \frac{5}{2}\sqrt{6\mu g d}$$

For elastic collision between two bodies

$$v_1 = \frac{(m_1 - m_2)u_1}{m_1 + m_2} + \frac{2m_2u_2}{m_1 + m_2}$$

Here $m_1 = m,\ m_2 = 4m,\ u_1 = 5\sqrt{6\mu g d},\ u_2 = 0$

$$\Rightarrow \quad v_1 = \frac{(m-4m)5\sqrt{6\mu g d}}{m+4m} + 0 = -3\times 5\frac{\sqrt{6\mu g d}}{5}$$

$$= -3\sqrt{6\mu g d}$$

The negative sign shows that the mass m rebounds. It then follows a projectile motion and its path's parabolic.

$u_y = 0,\ s_y = d,\ a_y = g,\ t_y = ?$

For vertical motion,

$$S = ut + \frac{1}{2}at^2 \Rightarrow d = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2d}{g}}$$

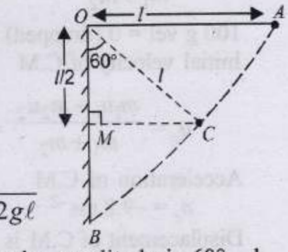
The horizontal distance travelled by mass m during this time t from P leftwards

$$x = 3\sqrt{6\mu g d} \times \sqrt{\frac{2d}{g}} = 6\sqrt{3\mu d^2} = \mathbf{6d\sqrt{3\mu}}$$

25. Initially when the bob of pendulum is at A, its P.E. = mgl

When the bob released from A and strikes to the wall at B, P.E. changes to K.E. and when it is at position 'C' the angular amplitude is 60°.

In ΔOCM



$$\cos 60° = \frac{OM}{\ell} \Rightarrow OM = \frac{\ell}{2}$$

The velocity of bob at B,

$$mg\ell = \frac{1}{2}mv_B^2 \quad \Rightarrow \quad v_B = \sqrt{2g\ell}$$

Let after n collisions, the angular amplitude is 60° when the bob again moves towards the wall from C, the velocity v'_B before collision

$$mg\frac{\ell}{2} = \frac{1}{2}mv'^2_B \quad \Rightarrow \quad v'_B = \sqrt{g\ell}$$

This means that the velocity of the bob should reduce from $\sqrt{2g\ell}$ to $\sqrt{g\ell}$ due to collisions with walls.

The final velocity after n collisions is $\sqrt{g\ell}$

$$\therefore \quad e^n(\sqrt{2g\ell}) = \sqrt{g\ell}$$

where e is coefficient of restitution.

$$\left(\frac{2}{\sqrt{5}}\right)^n \times \sqrt{2g\ell} = \sqrt{g\ell} \Rightarrow \left(\frac{2}{\sqrt{5}}\right)^n = \frac{1}{\sqrt{2}}$$

Taking log on both sides we get

$$n\log\left(\frac{2}{\sqrt{5}}\right) = \log\frac{1}{\sqrt{2}} \quad \Rightarrow \quad n = 3.1$$

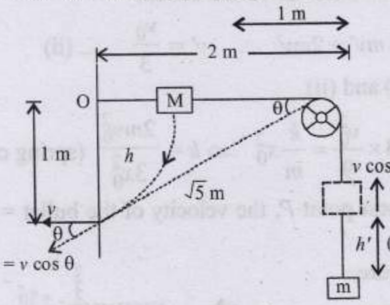
Hence, number of collisions = 4.

26. Let point mass hit the wall with the speed v.

Then, velocity of mass m at this instant $= v\cos\theta = \frac{2}{\sqrt{5}}v$.

Further M will fall a distance of 1 m while m will rise up by $(\sqrt{5}-1)$ m.

From energy conservation, Decrease in P.E. of M = increase in P.E. of m + increase in K.E. of both the blocks.



$$Mgh = mgh' + \frac{1}{2}mv^2 + \frac{1}{2}m(v\cos\theta)^2$$

or, $\quad (2)(9.8)(1) = (0.5)(9.8)(\sqrt{5}-1) + \frac{1}{2}\times 2\times v^2 + \frac{1}{2}\times 0.5\times\left(\frac{2v}{\sqrt{5}}\right)^2$

Solving we get, $v = 3.29$ m/s.

27. Initial position of C.M.

$$x_1 = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{100\times 0 + 100\times 98}{200} = 49\text{ m}$$

100 g vel = 0 (dropped)

Initial velocity of C.M

$$u_c = \frac{m_1u_1 + m_2u_2}{m_1 + m_2} = \frac{100\times 49 + 100\times 0}{200} = 24.5\text{ ms}^{-1}$$

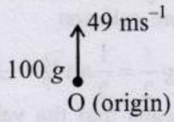
Acceleration of C.M

$a_c = -9.8\text{ ms}^{-2}$

Displacement of C.M is

$s_c = -49$ m

Applying



$$S = ut + \frac{1}{2}at^2$$

$-49 = 24.5\,t - 4.9\,t^2$

$t^2 - 5t - 10 = 0$

$$t = \frac{5\pm\sqrt{25+40}}{2} = \frac{5\pm\sqrt{65}}{2} = 6.53\text{ s}$$

28. The collision between C and A is elastic and their masses are equal so they will exchange their velocities A was initially at rest, therefore the result of collision will be that C will come to rest and A will initially start moving with a velocity v_0. But since A is connected to B with a spring, the spring will get compressed.

Let v' be the common velocity of A and B.

At $t = t_0$, the velocities of A and B become same.

Applying energy conservation;

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv'^2 + \frac{1}{2}2mv'^2 + \frac{1}{2}kx_0^2$$

where x_0 is the compression in the spring at $t = t_0$

$$\therefore\quad v_0^2 = 3v'^2 + \frac{k}{m}x_0^2 \qquad \text{...(i)}$$

Applying momentum conservation,

$$mv_0 = mv' + 2mv' \quad \therefore \quad v' = \frac{v_0}{3} \qquad \text{... (ii)}$$

From eq. (i) and (ii)

$$v_0^2 - 3\times\frac{v_0^2}{9} = \frac{k}{m}x_0^2 \quad\Rightarrow k = \frac{2mv_0^2}{3x_0^2} \text{ (spring constant)}$$

29. At the highest point P, the velocity of the bullet $= u\cos\theta$

In ΔAQR

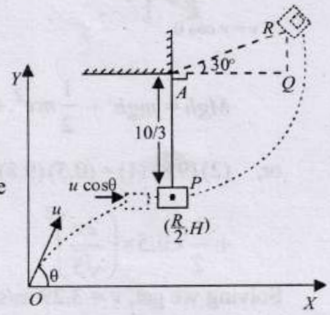
(i) From figure,

$$\sin 30° = \frac{QR}{10/3},\ QR = \frac{5}{3}$$

Now from, conservation of linear momentum at the highest point - P

$M(u\cos\theta) + 3M\times 0$

$= (M + 3M)\,v$



$$v = \frac{Mu\cos\theta}{4M} = \frac{u\cos\theta}{4}$$

From energy conservation principle *K.E.* of the bullet-mass system at P = *P.E.* of the bullet-mass system at R

$$\frac{1}{2}(4M)v^2 = (4M)\,gh$$

$$\frac{1}{2}(4M)\frac{u^2\cos^2\theta}{16} = 4Mg\times\left(\frac{10}{3} + \frac{5}{3}\right)$$

$$\cos^2\theta = \frac{9.8\times 5\times 2\times 16}{50\times 50} \Rightarrow \theta = 37°$$

($\because u = 50$ m/s given)

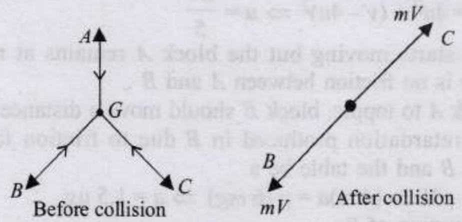
(ii) Vertical component,

$$\frac{R}{2} = \frac{u^2\sin 2\theta}{2g} = \frac{50\times 50\sin 2\times 37°}{2\times 9.8} = 122.6\text{ m}$$

Horizontal component,

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{50\times 50\times(\sin 37°)^2}{2\times 9.8} = 46\text{ m}$$

30.

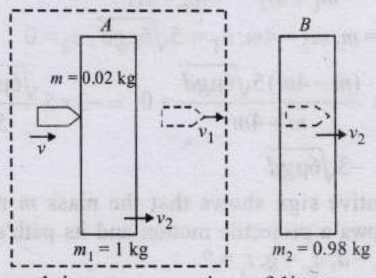


Before collision by symmetry, the net momentum of the system is zero.

After collision, momentum is also zero as no esternal force is acting on the system. A comes to rest so B and C should have equal and opposite momenta, so velocity of C should be same in magnitude *i.e.*, V but in opposite direction of velocity of B.

31. Applying conservation of linear momentum for the system of bullet and plate A, $mv = mv_1 + m_1v_2$

or, $\quad 0.02v = 0.02\,v_1 + 1\times v_2 \qquad$ ...(i)



Again applying conservation of linear momentum for collision at B, $mv_1 = (m + m_2)v_2$

or, $\quad 0.02\,v_1 = (2.98 + 0.02)\,v_2 = 3v_2$

$$\Rightarrow\quad v_2 = \frac{0.02v_1}{3} \qquad \text{... (ii)}$$

From eq. (i) and (ii)

$$0.02\,v = 0.02\,v_1 + \frac{0.02\,v_1}{3} \quad\Rightarrow\quad v = \frac{4}{3}v_1 \Rightarrow \frac{v}{v_1} = \frac{4}{3}$$

$$\frac{v_1}{v}=\frac{3}{4}\Rightarrow 1-\frac{v_1}{v}=1-\frac{3}{4}=\frac{1}{4}=0.25\Rightarrow\frac{v-v_1}{v}=0.25$$

$\therefore$ % loss in velocity, $\frac{v-v_1}{v}\times 100=\frac{1}{4}\times 100=25\%$

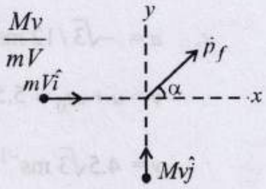
32. (i) According to consevation of linear momentum

$\vec{p}_f=\vec{p}_i=mV\hat{i}+Mv\hat{j}$ (Magnitude of the momentum of final body)

$\therefore\quad p_f=\sqrt{m^2V^2+M^2v^2}$

$\Rightarrow \tan\alpha=\frac{Mv}{mV}\Rightarrow\alpha=\tan^{-1}\frac{Mv}{mV}$

which gives the direction of the momentum.



(ii) $\frac{K.E_i-K.E_f}{K.E_i}=1-\frac{K.E_f}{K.E_i}=1-\frac{p_f^2(m+M)}{\frac{1}{2}mV^2+\frac{1}{2}Mv^2}$

$$=1-\frac{m^2V^2+M^2v^2}{(m+M)(mV^2+Mv^2)}$$

$$=\frac{m^2V^2+mMv^2+mMV^2+M^2v^2-m^2V^2-M^2v^2}{(m+M)(mV^2+Mv^2)}$$

$$\frac{\Delta K.E.}{K.E_i}=\frac{mM(v^2+V^2)}{(m+M)(mV^2+Mv^2)}$$

Topic-5: Miscellaneous (Mixed Concepts) Problems

1. **(6.30)** Given: mass $m=0.4$ kg; impulse J = 1.0 Ns $V_{(t)}=v_0e^{-t/\tau}$; $\tau=4$s and $e^{-1}=0.37$.

Impulse = Change in linear momentum

or, $J=mV_0\Rightarrow V_0=\frac{J}{m}=\frac{1}{0.4}=2.5\text{ ms}^{-1}$

Also $(V_t)=v_0e^{-t/\tau}\quad\therefore\ \frac{ds}{dt}=v_0e^{-t/\tau}\ \Rightarrow ds=v_0e^{-t/\tau}dt$

$\therefore s=v_0\int_0^\tau e^{-t/\tau}dt=v_0\tau(1-e^{-1})=2.5\times 4\times 0.63=6.30\text{ m}$

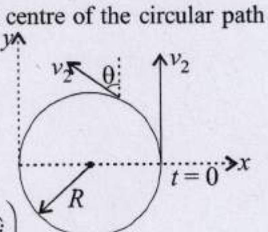
2. Angular speed of particle about centre of the circular path

$\omega=\frac{v_2}{R},\theta=\omega t=\frac{v_2}{R}t$

$v_p=\left(-v_2\sin\theta\hat{i}+v_2\cos\theta\hat{j}\right)$

or

$v_p=\left(-v_2\sin\frac{v_2}{R}t\hat{i}+v_2\cos\frac{v_2}{R}t\hat{j}\right)$



and $v_m=v_1\hat{j}$

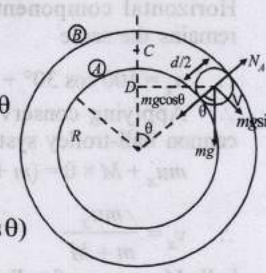
$\therefore$ Linear momentum of particle w.r.t. man as a function of time

$L_{pm}=m(v_p-v_m)$

$$=m\left[\left(-v_2\sin\frac{v_2}{R}t\right)\hat{i}+\left(v_2\cos\frac{v_2}{R}t-v_1\right)\hat{j}\right]$$

3. (a) For circular motion of the ball, the necessary centripetal force is provided by $(mg\cos\theta-N)$.

$mg\cos\theta-N=\frac{mv^2}{\left(R+\frac{d}{2}\right)}$ ... (i)



N = normal reaction at angle θ

According to energy conservation

$\frac{1}{2}mv^2=mg\left(R+\frac{d}{2}\right)(1-\cos\theta)$

$\Rightarrow\quad v^2=2g\left(R+\frac{d}{2}\right)(1-\cos\theta)$

Putting this value of v^2 in eq. (i)

$N=mg(3\cos\theta-2)$

(b) The ball will lose contact with the inner sphere when $N=0$

or $\quad 3\cos\theta-2=0$ or $\theta=\cos^{-1}\left(\frac{2}{3}\right)$

After this it makes contact with outer sphere and normal reaction starts acting towards the centre.

So for $\quad\theta\le\cos^{-1}\left(\frac{2}{3}\right)$

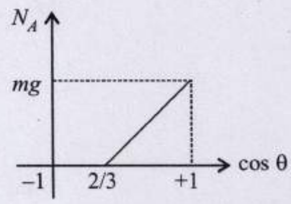
$N_B=0$

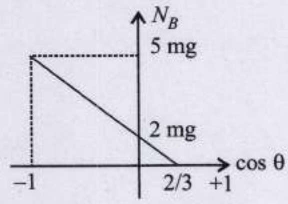
and $\quad N_A=mg(3\cos\theta-2)$

and for $\theta\ge\cos^{-1}\left(\frac{2}{3}\right)$

$N_A=0$ and $N_B=mg(2-3\cos\theta)$

The $N_A-\cos\theta$ and $N_B-\cos\theta$ graphs are as follows.



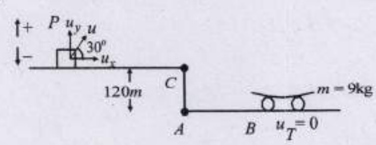


4. Consider the vertical motion of the cannon ball

$u_y=+100\sin 30°$ (vertical component of velocity)

$s_y=-120$ m (vertical displacement)

$a_y=-10\text{ m/s}^2$



$t=t_0$ (Time of first impact on carriage)

$s_y=u_yt+\frac{1}{2}a_yt_0^2\quad\therefore -120=50t_0-5t_0^2$

$\Rightarrow\quad t_0^2-10t_0-24=0$

$\therefore \quad t_0 = -\dfrac{(-10) \pm \sqrt{100 - 4(1)(-24)}}{2} = 12 \text{ or} -2$ [Not valid]

$\therefore \quad t_0 = 12$ sec.

Horizontal component of velocity of the cannon ball remains the same

$\therefore \quad u_x = 100 \cos 30° + 5\sqrt{3} = 55\sqrt{3}$

$\therefore$ Applying conservation of linear momentum to the cannon ball-trolley system in horizontal direction.

$mu_x + M \times 0 = (m + M)\, v_x$

$\therefore \quad v_x = \dfrac{mu_x}{m + M}$ (where m = mass of cannon ball, M = mass of trolley, v_x = velocity of the cannon ball-trolley system)

$\therefore \quad v_x = \dfrac{1 \times 55\sqrt{3}}{1 + 9} = 5.5\sqrt{3}\text{ ms}^{-1}$

Horizontal distance covered by the car

$P = 12 \times 5\sqrt{3} = 60\sqrt{3}$ m ($\because$ Second ball was projected after 12 second.)

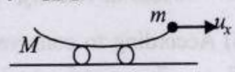
Since the second ball also struck the trolley,

$\therefore$ in time 12 seconds, the trolley covers a distance of $60\sqrt{3}$ m.

For trolley after 12 seconds;

$u = 5\sqrt{3}$ m/s, $v = ?$, $t = 12$ s

$s = 60\sqrt{3}$ m,



$s = ut + \dfrac{1}{2}at^2$

$\Rightarrow \quad 60\sqrt{3} = 5.5\sqrt{3} \times 12 + \dfrac{1}{2} \times a \times 144$

$\therefore \quad a = -\sqrt{3}/12 \text{ ms}^{-2} \quad \therefore v = u + at = 5\sqrt{3}$ m/s.

$v = u + at_0 = 5.5\sqrt{3} - \left(\dfrac{\sqrt{3}}{12}\right) \times 12$

$v = 4.5\sqrt{3}\text{ ms}^{-1}$

To find the final velocity v_f of the carriage after the second impact.

Applying conservation of linear momentum in the horizontal direction

$mu_x + (M + m)\, v_x = (M + 2m)\, v_f$

$1 \times 55\sqrt{3} + (9 + 1)\, 4.5\sqrt{3} = (9 + 2)\, v_f \therefore v_f = 15.75$ m/s